GCE

Further Mathematics A

Y534/01: Discrete Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless the question asks for a specific degree of accuracy.

Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$^{15}C_2 = 105$	M1 A1	1.1 1.1	$^{15}C_2$ or equivalent seen or implied 105 as final answer	If M0 scored then SC1 for 210 as final answer
	(b)		Some numbers can be made in more than one way, e.g. 1 and 11 give 111 both ways round	B1	2.4	Any suitable example that shows two different orders that give the same result.	e.g 1 followed by 13 or 11 followed by 3 both give 113
	(c)		Lead digit is 1 when first card is 1 or any of 10, 11, 12, 13, 14, 15.	B1	2.4	Identifying that double digit cards also start with 1 7 of the cards have lead digit 1	Benford's law BOD 6 or 8
	(d)		He does not need to actually find these primes, only find out how many there are.	B1	2.4	It is an enumeration problem	Constructing some solutions would not necessarily give a full count
				[5]			

2	(a)		<table><tr><td>Vertex</td><td>J</td><td>K</td><td>L</td><td>M</td><td>N</td></tr><tr><td>Degree</td><td>1</td><td>3</td><td>2</td><td>2</td><td>2</td></tr></table> <table><tr><td>Vertex</td><td>P</td><td>Q</td><td>R</td><td>S</td><td>T</td></tr><tr><td>Degree</td><td>3</td><td>2</td><td>2</td><td>2</td><td>1</td></tr></table>	M1 M1 A1	1.1 1.1 1.1	Each graph has at least three vertices of degree 2 listed J and T each have degree 1 All correct	
	(b)		e.g. In G1, the vertex with degree 3 is connected to the vertex of degree 1 (and two of degree 2) In G2, the vertex of degree 3 is connected to three vertices of degree 2 e.g. In G1 the vertex of degree 1 is connected to the vertex of degree 3 but in G2 it is connected to a vertex of degree 2.	B1 B1	1.1 2.1	The region in G1 has four edges (regions have four edges) The region in G2 has three edges (regions have three edges) Or equivalent <u>written</u> explanation	Incomplete explanation that uses both graphs = B1, B0 e.g. No arc between J and N but P is joined to Q
	(c)	(i)	Still (at least one) odd degree	B1	2.1	Still have vertices of degree 3	P and/or K not even degrees
		(ii)	Add an arc joining P to K	B1	1.1	P to K, written	Arc PK

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th><th></th></tr>
<tr><td></td><td>(d)</td><td></td><td>Each of the 3 vertices in K_3 must be connected to each of the 4 vertices in K_4.

3×4 = 12</td><td>B1
B1

B1</td><td>2.1
1.1

1.1</td><td>Appropriate written explanation
K_3 has 3 vertices, K_4 has 4 vertices (or K_n has n vertices)
Appropriate calculation shown (leading to the value 12, as given)</td><td>K_7 has 21 arcs
K_3 has 3 arcs and K_4 has 6 arcs
21 – (3 + 6) = 12)</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[10]</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>(a)</td><td></td><td>Shuttle (sort)</td><td>B1</td><td>1.2</td><td>Or insertion</td><td></td></tr>
<tr><td></td><td>(b)</td><td></td><td><table><tr><th>Input list</th><th>L</th><th>N</th><th>Output list</th></tr><tr><td>5 7 6 4</td><td>6</td><td>9</td><td>6</td></tr><tr><td>7 6 4</td><td>6</td><td>5</td><td>6 9</td></tr><tr><td>6 4</td><td>5</td><td>7</td><td>5 6 9</td></tr><tr><td>6 4</td><td>6</td><td>7</td><td>5 6 9</td></tr><tr><td>6 4</td><td>9</td><td>7</td><td>5 6 9</td></tr><tr><td>4</td><td>5</td><td>6</td><td>5 6 7 9</td></tr><tr><td>4</td><td>6</td><td>6</td><td>5 6 7 9</td></tr><tr><td></td><td>5</td><td>4</td><td>5 6 6 7 9</td></tr><tr><td></td><td></td><td></td><td>4 5 6 6 7 9</td></tr></table></td><td>B1
B1
B1
B1

B1</td><td>1.1
1.1
1.1
1.1

1.1</td><td>ANSWER SPACE IS ON 2 PAGES
First row correct in all four columns
L = 6, N = 9 and then N = 5
L = 5, 6, 9 (in this order) with N = 7
L = 5, 6 (in this order) with N = 6

{input list} ∪ {N} ∪ {output list} = {4, 5, 6, 6, 7, 9} in at least 4 different rows</td><td>Treat any blank cells with values on that column in a row above and a row below as rolling down from the row above</td></tr>
<tr><td></td><td>(c)</td><td>(i)</td><td>8</td><td>B1ft</td><td>1.1</td><td>Correct or follow through (b)</td><td>Number of different rows, excluding ‘print’ row</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>15</td><td>B1</td><td>2.2b</td><td>1+2+3+4+5</td><td></td></tr>
<tr><td></td><td>(d)</td><td></td><td>Sorting is O(n^2) or quadratic order

$15\times\left(\frac{300}{60}\right)^2$

= 375 seconds</td><td>M1

M1

A1</td><td>1.1

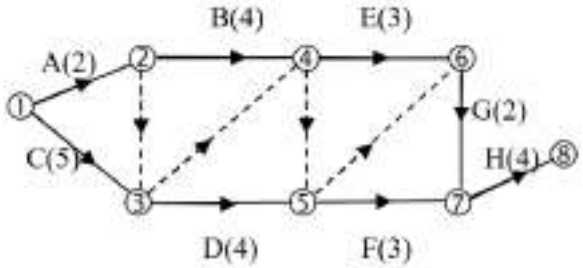
1.1

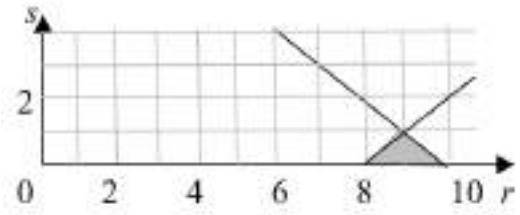
2.2b</td><td>Seen or implied from working

Size of problem is $\times\frac{300}{60}$ = ×5
Or $k \times 60^2 = 15$ so $k = \frac{1}{240}$
cao, with units</td><td>Allow M1 for $\frac{1}{2}n(n-1)$ or any quadratic function, algebraic

M0, M1 for treating problem as O(n) or O(n^3)

Or 6 minutes 15 seconds</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[11]</td><td></td><td></td><td></td></tr>
</table>

Question			Answer	Marks	AO	Guidance	
4	(a)			**M1** **A1**	**3.1a** **1.1**	ANSWER SPACE IS ON 2 PAGES Activity on arc network with 8 activities, A to H Correct, with directed arcs and no extra dummies. Durations need not be shown	
			<table><tr><td>Event</td><td>①</td><td>②</td><td>③</td><td>④</td><td>⑤</td><td>⑥</td><td>⑦</td><td>⑧</td></tr><tr><td>Early time</td><td>0</td><td>2</td><td>5</td><td>6</td><td>9</td><td>9</td><td>12</td><td>16</td></tr><tr><td>Late time</td><td>0</td><td>3</td><td>5</td><td>7</td><td>9</td><td>10</td><td>12</td><td>16</td></tr></table> Longest path C – D – F – H	**M1** **A1ft** **B1**	**1.1** **1.1** **3.2a**	Forward pass attempted Forward and backward passes correct, follow through their activity on arc network with at least one burst and at least one merge Critical path correct	M1 may be implied from 16 as length of longest path (www) or from correct longest path Or equivalent description
			<table><tr><td>Event</td><td>①</td><td>②</td><td>③</td><td>④</td><td>⑤</td><td>⑥</td><td>⑦</td><td>⑧</td></tr><tr><td>Temporary</td><td></td><td>2</td><td>5 2</td><td>6 2</td><td>6 2</td><td>5 2</td><td>5 4</td><td>8</td></tr><tr><td>Permanent</td><td>0</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>4</td><td>8</td></tr></table> Shortest path A – G - H	**M1** **A1ft** **B1**	**1.1** **1.1** **3.2a**	Evidence of Dijkstra (at vertices) Follow through their network with at least one vertex updated Shortest route correct	M1 may be implied from 8 as length of shortest path (www) or from correct shortest path Or equivalent description
	(b)		Minimum completion time	**B1**	**3.2a**	Or equivalent	Referring to time for critical activities (or equivalent)
	©		1	**B1**	**3.2a**	Units may be implied	
				[10]			

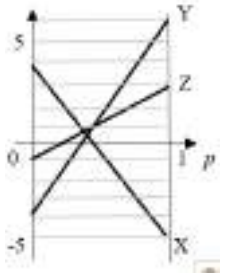
Question			Answer	Marks	AO	Guidance	
5	(a)		g, r, s = time spent on gym work, running, swimming, respectively, in hours	B1	1.1	Define variables	Or equivalent with other letters e.g. x, y, z
			Maximise $3g + r + 1.5s$ subject to	B1	3.3	Objective (maximise and positive coefficients in ratio 6:2:3)	Max $g + \frac{1}{3}r + \frac{1}{2}s$ NOT Max $g + 3r + 2s$
			$g + r + s \leq 18$	B1	1.1	$g + r + s \leq 18$	
			$g \leq 8$ $r \geq 4$	B1	3.3	$g + s \leq r$	Ignore extra constraints provided they are not inconsistent
			$g + s \leq r$ $g \geq 0$ and $s \geq 0$	B1	1.1	$0 \leq g \leq 8$, $s \geq 0$ and $r \geq 4$	
	(b)	(i)	Set $g = 8$: Maximise $r + 1.5s$ (+24) subject to $r + s \leq 10$, $s + 8 \leq r$ and $s \geq 0$	B1	3.1b	ANSWER SPACE IS ON 2 PAGES Reformulate as a 2-variable problem with an objective function and at least two non-trivial constraints	A linear expression or linear equation and at least two non-trivial constraints in their r and s
			e.g.  	M1	1.1	A finite FR identified on graph with variables their r and s	
				A1	3.4	Correct FR identified	
			Optimal at $r = 9$, $s = 1$	M1	1.1	A vertex where 2 of their lines (with variables their r and s) cross	M1 may be implied from correct solution in context
			Spend 9 hours running and 1 hour swimming.	A1	3.4	Interpret their solution (with non-negative times) in context	
		(ii)	e.g. would need to rest/eat/sleep e.g. need transition time between activities	B1	3.5b	A reason <u>why</u> their solution is not practical	e.g. should not train for 18 hours just before a race
		(iii)	e.g. build in recovery time e.g. for each hour allocated to each type of training only 50 minutes is spent on the training.	B1	3.5c	Describing <u>how</u> model can be adapted to make their solution more realistic (do not need specific times)	e.g. restrict the preparation time to something that is less than 18 hours
				[12]			

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AO</th><th>Guidance</th><th></th></tr>
<tr><td>6</td><td>(a)</td><td></td><td>Total pay-off is 16, subtract 8 from each pay-off.
Emma
Drew
P: -5, 6, 3
Q: 4, -4, -1
R: 3, -4, -2</td><td>M1

A1</td><td>1.1

1.1</td><td>Converting to zero-sum, or implied from total = 16 (for each cell) or from working
Pay-off matrix for Drew all correct (or a positive multiple of this)</td><td>Each player puts 8 into ‘pot’
May use differences
P: -10, 12, 6
Q: 8, -8, -2
R: 6, -8, -4</td></tr>
<tr><td></td><td>(b)</td><td></td><td>Strategy Q (weakly) dominates strategy R
so matrix can be reduced
Emma
Drew
P: -5, 6, 3
Q: 4, -4, -1

Let Drew choose randomly between P and Q so that the probability of P is p and the probability of Q is $1 - p$.

If Emma plays X, Drew expects $-5p + 4(1 - p)$
If Emma plays Y, Drew expects $6p - 4(1 - p)$
If Emma plays Z, Drew expects $3p - (1 - p)$
 

$4 - 9p = 10p - 4$
$p = \frac{8}{19}$
P with probability $\frac{8}{19}$ and Q with probability $\frac{11}{19}$</td><td>
M1

A1

B1

M1

A1</td><td>
1.1

2.1

1.1

1.1

2.1</td><td>ANSWER SPACE IS ON 2 PAGES
Identifying dominance
Row (from their zero-sum table) removed by dominance

Describing (prob of) P = p and (prob of) $Q = 1 - p$, or equivalent, in terms of <u>one</u> variable only

Or $4 - 9p$
Or $10p - 4$
Or $4p - 1$
All three correct (in any form) for the values in their 2×3 reduced table
Or implied from graph if no expressions are given

Solving simultaneous equations when X = Y
Both probabilities correct in context</td><td>If an A-level candidate chooses to use simplex credit as below:

M1: Add 5 to make all entries non-negative, set up initial tableau
A1: Initial tableau correct

B1: At least two valid iterations
M1 Achieving final tableau with p = $\frac{8}{19}$, q = $\frac{11}{19}$, r = 0
A1 Correct interpretation in context.

Check with TL if necessary</td></tr>
</table>

Question			Answer	Marks	AO	Guidance	
	(c)		From graph, if Drew plays optimally then Emma should not choose Z.	M1	2.2a	Or equivalent	
			Let Emma choose randomly between X and Y so that the probability of X is x and the probability of Y is $1 - x$.	A1	2.1	Description including symbols used for probabilities (one unknown only)	
			$5x - 6(1 - x) = -4x + 4(1 - x)$ or $= -\frac{4}{19}$	B1	1.1	Pay-offs for Emma are × (–1)	
				M1	1.1	A valid method	
			X with probability $\frac{10}{19}$ and Y with probability $\frac{9}{19}$	A1	2.1	Either probability correct	
				[12]			

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